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Top quark effects on the scalar sector of the minimal Standard Model

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Abstract

We study effects of the heavy top quark ($m_t \approx 180$GeV) on the scalar sector of the minimal Standard Model. We construct the effective potential for the scalar doublet, by first taking into account the leading contributions of the top quark loops. Minimizing this potential gives us a condition analogous to the leading-N_c gap equation of the standard $\langle \bar{t}t \rangle$-condensation model (Top-mode Standard model). This essentially non-perturbative condition leads to a low ultraviolet cut-off $\Lambda = \mathcal{O}(1 \text{ TeV})$ in the case when the bare mass μ of the scalar doublet in the tree-level potential satisfies $\mu^2 = -M_0^2 \leq 0$ and the scalar doublet there is self-interacting ($\lambda > 0$). We demand that the scalar self-interaction behave perturbatively – in the sense that its 1-loop contributions influence the effective potential distinctly less than those of the Yukawa coupling of the heavy top quark. When we subsequently include the 1-loop contributions of the scalar and the gauge bosonic sectors in a perturbative manner, the results change numerically, but the cut-off Λ remains $\mathcal{O}(1 \text{ TeV})$. The resulting Higgs mass M_H is then in the range 150-250 GeV. Furthermore, the results of the paper survive even in the case when the square of the bare mass μ^2 is positive, as long as $\mu^2 \leq \mathcal{O}(\lambda v^2)$, where $\lambda/4!$ is the usual bare coupling parameter of the quartic self-interaction of the scalars and v is the vacuum expectation value ($v = 246.2$ GeV, $M_H^2 \approx \lambda v^2/3$).

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1 Introduction

In the minimal Standard Model (MSM), the scalar sector is probably the most misterious one. Experiments haven't yet shown any direct evidence of the Higgs. The indirect evidence is very hard to pinpoint, since the results of the measurements up to date either do not depend or depend very mildly on the scalar structure of the model and on the Higgs mass. Therefore, the mass of the Higgs of the MSM is still largely unknown (65 GeV $< M_H \lesssim$ 0.8TeV) [1]. The main reason for having the scalar sector in the Standard Model is to have a viable mechanism to induce an electroweak spontaneous symmetry breaking (SSB) leading to the phenomenologically required masses of the electroweak gauge bosons W and Z. While SSB in this scenario is directly related to parameters of the scalar sector, these can be dramatically influenced by effects of other sectors on the scalar one.

The experimental evidence of a heavy top quark ($m_t \approx 180$ GeV) [2] suggests that the resulting strong Yukawa coupling parameter $g_t \approx 1$ may drastically influence the scalar sector, through quantum loops. These effects on the parameters of the effective potential of the scalar doublet $V_{\text{eff}}(\Phi)$ may have a non-perturbative nature. For example, they may be responsible for inducing SSB. To treat such non-perturbative effects with diagrammatic (loop) approach is legitimate, because these are effects of one (quark) sector of the model on another (scalar) sector. On the other hand, the diagrammatic treatment of the quantum effects of the scalar self-interaction on the scalar sector itself has predictive power only if these effects are reasonably weak – of perturbative nature.

In section II, we investigate the leading effects of the heavy top quark sector on the scalar sector, by calculating the contributions of the top quark loop to the effective potential $V_{\text{eff}}(\Phi)$. Minimizing this potential, we end up with a relation connecting the bare parameters of the scalar sector [1] to the ultraviolet cut-off, the vacuum expectation value and m_t. This relation is analogous to the leading-N_c gap equation in a simple model of the $\langle \bar{t}t \rangle$-condensation (Top-mode Standard Model - TSM [3]). From this relation we infer that the cut-off Λ of the theory should have a stringent upper bound, if the square of the bare mass μ^2 of the scalar doublet is non-positive [2] or at least smaller than $\mathcal{O}(\lambda v^2)$, where $\lambda/4!$ is the bare coupling parameter of the quartic scalar self-interaction ($\lambda \geq 0$), and v =246 GeV. Under these conditions, we obtain the upper bound $\Lambda^{\text{u.b.}}$ and the Higgs mass M_H as a function of the parameter λ. For $\lambda \lessapprox 3$, we obtain $\Lambda^{\text{u.b.}} = \mathcal{O}(1 \text{ TeV})$ and M_H in the region of 150-250 GeV. If, however, μ^2 is positive *and* surpasses $\lambda v^2/6$, the cut-off Λ of the theory becomes less restricted and values $\Lambda \geq \mathcal{O}(10 \text{ TeV})$ become possible. In such a case, since $\mu^2 \geq 0$, we do not have SSB at the tree

[1] i.e, the bare parameters *before* the inclusion of the top quark effects.

[2] Note that for negative μ^2 ($\mu^2 \neq 0$) we have SSB already at the tree level.

level.

In Section III, we include in a perturbative manner the 1-loop effects of the scalar self-interactions, assuming them not to be too strong. By this we mean that they are distinctly smaller than the heavy top quark effects calculated in Section II. This in turn implies that the value of the bare coupling parameter λ cannot be larger than about 3. We also include the 1-loop effects of the gauge bosons. The latter effects are substantially smaller than the contributions of the heavy quark sector, and are hence also calculated in a perturbative manner. The modified numerical results are presented in Table 1. It turns out that the qualitative features of Section II survive, i.e., $\Lambda = \mathcal{O}(1 \text{ TeV})$ and $M_H = 150 - 250$ GeV.

For the results of the present paper, it was important to treat the cut-off Λ of the model (MSM) as a finite and physical quantity, i.e., Λ is roughly the energy where the MSM is replaced by some new, as yet unknown, physics. For simplicity, we were using simple covariant spherical cut-off for the Euclidean 4-momenta of the loops (after the Wick rotation). The presence of Λ^2 and $\ln(\Lambda/m_t)$-terms in the effective potential was crucial in our analysis. We neglected terms of $\mathcal{O}(\Lambda^0)$. The possible errors resulting from this and other approximations were estimated toward the end of the paper. We emphasize that the terms $\mathcal{O}(\Lambda^0)$ in our integrals depend on the regularization (cut-off) procedure chosen.

We stress that the present work was largely motivated by the work of Fatelo *et al.* [4] and the ideas contained therein. They discussed the case of the zero bare coupling λ. The present work can be regarded as an extension of their work to the case of nonzero values of the bare coupling λ.

The basic result of the present paper is the following: in a substantial part of the parameter space of the bare scalar couplings, the interplay of the heavy top and the scalar sector leads to the conclusion that the MSM is replaced by some new physics at a relatively low scale $\mathcal{O}(1 \text{ TeV})$. In other parts of that parameter space, much larger cut-offs are still possible. The results are valid and predictable as long as the scalar sector is not too strongly self-interacting, i.e., as long as the Higgs mass is in the region 150-250 GeV.

2 Top quark (non-perturbative) contributions to the effective potential

We start with the following Lagrangian, containing only the sectors of our primary interest – the scalar and the quark sector, assuming that the only non-zero Yukawa coupling is g_t:

$$\begin{aligned}\mathcal{L}^{(\Lambda)} &= i\bar{t^a}\partial\!\!\!/ t_a + i\bar{b^a}\partial\!\!\!/ b_a + \partial_\mu\Phi\partial^\mu\Phi^\dagger - V^{(0)}\left(2\Phi\Phi^\dagger;\Lambda\right) \\ &\quad -\frac{g_t(\Lambda)}{\sqrt{2}}\bar{t^a}\left(\varphi - i\gamma_5 G^{(0)}\right)t_a + \frac{g_t(\Lambda)}{2}\left[G^{(+)}\bar{t^a}(1-\gamma_5)b_a + G^{(-)}\bar{b^a}(1+\gamma_5)t_a\right] ,\end{aligned} \tag{1}$$

where we took $g_b(\Lambda) \approx 0$, and Φ is the scalar $SU(2)_L$-doublet of the MSM

$$\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix}\sqrt{2}G^{(+)} \\ \varphi + iG^{(0)}\end{pmatrix} . \tag{2}$$

Here, φ, $G^{(0)}$ and $G^{(\pm)}$ are the Higgs field (before the symmetry breaking) and the neutral and the charged Goldstone fields, respectively. In (1), Λ is the effective cut-off of the theory, a is the color index for the top quark, and $V^{(0)}$ denotes the tree level potential

$$V^{(0)}\left(\varphi^2;\Lambda\right) = -\frac{1}{2}M^2(\Lambda)\varphi^2 + \frac{\lambda(\Lambda)}{4!}\varphi^4 . \tag{3}$$

Here, and from now on, we denote in the effective potentials explicitly only the dependence on the square φ^2 of the unbroken Higgs field. The general $SU(2)_L$-invariant expressions are obtained always by simply substituting $\varphi^2 \mapsto 2\Phi\Phi^\dagger$, thus explicitly including the Goldstone degrees of freedom. In (3), $M^2(\Lambda) = -\mu^2(\Lambda)$, where $\mu(\Lambda)$ is formally the bare mass of the scalars before the electrowek symmetry breaking; $\lambda(\Lambda)$ is the non-negative bare parameter of the quartic self-coupling term. For simplicity, we always omit the superscript Λ in the bare fields: $\varphi^{(\Lambda)} \mapsto \varphi$. In this Section, where we only consider the leading influence of the heavy top quark on the scalar sector, the Goldstone degrees of freedom will not play any direct role.

It is well known that in the case of "imaginary" masses μ, i.e., $M^2(\Lambda) > 0$, we have spontaneous symmetry breaking (SSB) already at the tree level: $\langle\varphi\rangle = \pm\sqrt{6M^2(\Lambda)/\lambda(\Lambda)}$ and $M_H^2(\Lambda) = \lambda(\Lambda)\langle\varphi\rangle^2/3$, where $\langle\varphi\rangle$ means the vacuum expectation value (VEV), and M_H is the mass of the Higgs $H = \varphi - \langle\varphi\rangle$.

The heavy top quark contributes at 1-loop level appreciably to the effective potential of the Higgs, because of the strong Yukawa coupling $g_t(\Lambda) \sim 1$. These contributions can be obtained, for example, by simply calculating the truncated Green functions $\tilde{\Gamma}_\varphi^{(2n)}(p_1,\ldots,p_{2n})$ at zero external

momenta, corresponding to the diagrams of Figs. 1a-c

$$V_{\rm eff}\left(\varphi^2;\Lambda\right) = V^{(0)}\left(\varphi^2;\Lambda\right) + i\sum_{n=1}^{\infty}\frac{1}{(2n)!}\tilde{\Gamma}_{\varphi}^{(2n)}\left(p_1,\ldots,p_{2n}\right)\Big|_{(\{p_k\}=0)}\varphi^{2n} \ . \tag{4}$$

This was calculated, for example, in ref. [5], for the case of a scalar field which is initially unbroken and non-dynamical (auxiliary), within a $\langle\bar{t}t\rangle$-condensation framework (Top-mode Standard Model – TSM). It is straightforward to see that the question whether the scalar φ is dynamical or non-dynamical at the outset, does not play any role for the truncated Green functions corresponding to Figs. 1a-c. Furthermore, we have explicitly checked that the 1-loop contribution of the top quark to $V_{\rm eff}$ remains the same even when the field φ is broken already at the tree level [3], i.e., when $M^2(\Lambda) > 0$. Therefore, we can simply copy the result of ref. [5], replacing there the formal Yukawa coupling parameter $g = M_0\sqrt{G}$ by the actual bare Yukawa coupling parameter $g_t(\Lambda)$

$$\begin{aligned} V_{\rm eff}\left(\varphi^2;\Lambda\right) &= V^{(0)}\left(\varphi^2;\Lambda\right) + V^{(1\ell t)}\left(\varphi^2;\Lambda\right) \\ &= V^{(0)}\left(\varphi^2;\Lambda\right) - \frac{N_c}{8\pi^2}\int_0^{\Lambda^2} d\bar{k}^2\bar{k}^2\ln\left[1+\frac{g_t^2(\Lambda)\varphi^2}{2\bar{k}^2}\right] \ . \end{aligned} \tag{5}$$

We see that the 1-loop top quark (superscript: $1\ell t$) contribution is proportional to the number of colors $N_c = 3$, since each color contributes independently to the loops of Figs. 1a-c. For the integral (5), Wick rotation has been performed and the integral is written in the Euclidean metric, $\bar{k}$ is the Euclidean loop momentum of the top quark. For simplicity, we used covariant spherical cut-off.

The minimum of $V_{\rm eff}$ is achieved at the value of φ that is the vacuum expectation value (VEV) $\langle\varphi\rangle_{1\ell t}$, where the subscript denotes that this is an approximation with only the leading (1-loop) heavy top quark quantum effects taken into account. We will call the relation resulting from this minimum the "gap" equation [4]

$$\frac{\partial V_{\rm eff}^{(0+1\ell t)}\left(\varphi^2;\Lambda\right)}{\partial\varphi^2}\Bigg|_{\varphi=\langle\varphi\rangle_{1\ell t}} = 0 \qquad \Longrightarrow$$

$$\kappa(\Lambda)\left[\Lambda^2 - m_t^{(0)2}(\Lambda)\ln\left(\frac{\Lambda^2}{m_t^{(0)2}(\Lambda)}+1\right)\right] = \frac{\lambda(\Lambda)}{12}\langle\varphi\rangle_{1\ell t}^2 - \frac{1}{2}M^2(\Lambda) \ , \tag{6}$$

where we denoted

$$\kappa(\Lambda) = \frac{g_t^2(\Lambda)N_{\rm c}}{16\pi^2} \ , \qquad m_t^{(0)}(\Lambda) = \frac{g_t(\Lambda)\langle\varphi\rangle_{1\ell t}}{\sqrt{2}} \ . \tag{7}$$

In the present framework, we should regard $m_t^{(0)}(\Lambda)$ as the running mass of the top quark at the energy of the upper cut-off $E = \Lambda$, i.e., the bare mass of the top quark. Later in this Section we will show that

[3] This conclusion could be arrived at also by invoking $SU(2)_L$-symmetry arguments.

[4] We choose this term in analogy with the terminology of the $\langle\bar{t}t\rangle$-condensation mechanism.

this mass is approximately equal, apart from small radiative corrections, to the physical mass $m_t^{\rm phy}$. Furthermore, we can regard, in the present framework, $\langle\varphi\rangle_{1\ell t}$ as the actual VEV v (= 246.2 GeV), apart from small radiative corrections that will be accounted for later.

Note that the Λ^2-term plays a crucial role in this relation, similarly as it does also in the $\langle\bar{t}t\rangle$-condensation mechanism. Formally, the only difference in (6) from the usual leading-$N_{\rm c}$ gap equation of the TSM-type $\langle\bar{t}t\rangle$-condensation is the presence of the $\lambda(\Lambda)$-terms, due to the self-interaction of the scalar sector. The "gap" equation (6) can be regarded as representing a perturbative influence of the top quark sector on the scalar sector only if the resulting VEV $\langle\varphi\rangle_{1\ell t}$ is relatively close to the tree level VEV $\langle\varphi\rangle_0 = \sqrt{6M^2(\Lambda)/\lambda(\Lambda)}$. Otherwise, the relation (6) should be regarded as an inherently non-perturbative effect of the top sector on $V_{\rm eff}$. For example, for $M^2(\Lambda) \approx 0$, i.e., $\langle\varphi\rangle_0 \ll 246$ GeV, the effects are highly non-perturbative. The above "gap" equation can be rewritten as

$$2\kappa(\Lambda)\frac{\Lambda^2}{m^2(\Lambda)} = \left[1 - z_1 \ln\left(z_1^{-1}+1\right)\right]^{-1} \qquad \left(\gtrapprox\ 1\right)\ , \tag{8}$$

$$\text{where we denote:}\quad m^2(\Lambda) = \frac{\lambda(\Lambda)}{6}\langle\varphi\rangle_{1\ell t}^2 - M^2(\Lambda)\ , \qquad z_1 = \frac{m_t^{(0)2}(\Lambda)}{\Lambda^2} \quad (\ <\ 1)\ . \tag{9}$$

We have $m^2(\Lambda) > 0$, which means that the following relation between the bare parameters $M^2(\Lambda)$ and $\lambda(\Lambda)$ of the starting scalar potential and the solution $\langle\varphi\rangle_{1\ell t}^2$ is automatically fulfilled

$$M^2(\Lambda) < \lambda(\Lambda)\langle\varphi\rangle_{1\ell t}^2/6\ . \tag{10}$$

In the case of the tree-level SSB ($M^2(\Lambda) > 0$), we then have by (6)

$$\langle\varphi\rangle_{1\ell t}^2 - \langle\varphi\rangle_0^2 = 12\frac{\kappa(\Lambda)}{\lambda(\Lambda)}\Lambda^2\left[1 - z_1\ln(z_1^{-1}+1)\right]\ \lessapprox\ 12\frac{\kappa(\Lambda)}{\lambda(\Lambda)}\Lambda^2\ , \tag{11}$$

where $\langle\varphi\rangle_0 = \sqrt{6M^2(\Lambda)/\lambda(\Lambda)}$ is the VEV at the tree level. We see that the 1-loop top quark effects, in the case of the tree-level SSB, increase the square of the VEV by a term roughly proportional to Λ^2.

If, on the other hand, $M^2(\Lambda)$ is non-positive, we have no SSB at the tree level, but we do have it after the inclusion of the top quark effects – i.e., the quantum-induced SSB according to (6).

The central question appearing here is: can we obtain any restrictions on the cut-off Λ from the heavy-top-influenced "gap" equation (6)? And, what are the resulting masses of the Higgs? The relations (6) and (8) can be rewritten

$$\Lambda^2 = \frac{m^2(\Lambda)}{2\kappa(\Lambda)\,(1-\theta(\Lambda))}\ \lessapprox\ \frac{\lambda(\Lambda)\langle\varphi\rangle_{1\ell t}^2}{12\kappa(\Lambda)\,(1-\theta(\Lambda))}\ , \tag{12}$$

$$\text{where} \qquad \theta(\Lambda) = \frac{m_t^{(0)2}(\Lambda)}{\Lambda^2} \ln \frac{\Lambda^2}{m_t^{(0)2}(\Lambda)} + \mathcal{O}\left(\frac{m_t^4}{\Lambda^4} \right) \ , \tag{13}$$

and the inequality $\lessapprox$ in (12) holds when either $M^2(\Lambda) \geq 0$, or $M^2(\Lambda)$ is negative and satisfying $-M^2(\Lambda) = |M^2(\Lambda)| \ll \lambda(\Lambda) \langle \varphi \rangle_{1\ell t}^2 / 6$. The parameters appearing in this upper bound are bare parameters – at the "running" energy Λ of the ultimate ultraviolet cut-off of the theory. We would like to express this upper bound for the cut-off Λ in terms of renormalized parameters, i.e., in terms of physical quantities which are, with the exception of the physical Higgs mass M_H, reasonably well known. Therefore, we devote the next few paragraphs to the relations between the bare parameters appearing in (12) and the physical parameters, within the present framework of including only the 1-loop effects of the heavy quark.

The relation between the physical (pole) mass $M_H^{\rm pole}$ and the corresponding bare coupling $\lambda(\Lambda)$, as well as the relation between the physical VEV $\langle \varphi_{\rm ren.} \rangle = v$ and the bare VEV $\langle \varphi \rangle$, in the present framework of included 1-loop top quark effects only, are expressed by means of the following truncated Green function, corresponding to the diagram of Fig. 2a

$$- i \Sigma_{HH}(q^2) = -i \Sigma_{HH}^{tt}(q^2) = \left(\frac{g_t(\Lambda)}{\sqrt{2}} \right)^2 N_{\rm c} \int \frac{d^4 k}{(2\pi)^4} tr_f \left[\frac{i}{\left(\not{k} - m_t^{(0)}(\Lambda) \right)} \frac{i}{\left(\not{k} + \not{q} - m_t^{(0)}(\Lambda) \right)} \right] \ . \tag{14}$$

These relations are

$$\begin{aligned} \left(M_H^{\rm pole} \right)^2 &= \frac{d^2 V^{(0)}}{d\varphi^2} \Big|_{\varphi = \langle \varphi \rangle_{1\ell t}} + \Sigma_{HH} \left(q^2 = \left(M_H^{\rm pole} \right)^2 \right) \\ &= \frac{d^2 V_{\rm eff}^{(0+1\ell t)}}{d\varphi^2} \Big|_{\varphi = \langle \varphi \rangle_{1\ell t}} + \Sigma_{HH} \left(q^2 = \left(M_H^{\rm pole} \right)^2 \right) - \Sigma_{HH} \left(q^2 = 0 \right) \ , \end{aligned} \tag{15}$$

$$Z_\varphi \varphi^2 = \varphi_{\rm ren.}^2 \ , \qquad \text{where: } Z_\varphi = \left[1 - \frac{d\Sigma_{HH}(q^2)}{dq^2} \Big|_{q^2 = (M_H^{pole})^2} \right] = 1 + \delta Z_\varphi \ . \tag{16}$$

In particular, the renormalized VEV $v = \langle \varphi_{\rm ren.} \rangle$ ($= 246.2$ GeV) is related to the bare VEV $\langle \varphi \rangle$ ($= \langle \varphi \rangle_{1\ell t}$ in this Section) by

$$\langle \varphi \rangle^2 = (1 - \delta Z_\varphi) \, v^2 \ . \tag{17}$$

These relations can be obtained by simply summing up in the Higgs propagator the $t\bar{t}$-loops of Fig. 2a in the leading-log approximation (geometrical series), starting with the "bare" Higgs propagator with the mass equal to the $d^2 V^{(0)} / d\varphi^2$ evaluated at the *corrected* [5] VEV $\langle \varphi \rangle_{1\ell t}$. Σ_{HH}^{tt} can be calculated

[5] It is at this corrected VEV that the linear term ($\propto H = \varphi - \langle \varphi \rangle$) of the tree level potential $V^{(0)}$ is canceled by the 1-loop top quark tadpole.

directly, by performing first the Wick rotation in the Euclidean space. Then we impose again the spherical cut-off Λ on the Euclidean top quark momentum $\bar{k}$ and end up with the following result

$$\Sigma_{HH}^{tt}\left(q^2\right) = -2\kappa(\Lambda)\Big\{\Lambda^2 + \left[\frac{q^2}{2} - 3m_t^{(0)2}(\Lambda)\right]\ln\left(\frac{\Lambda^2}{m_t^{(0)2}(\Lambda)}\right) + \mathcal{O}(q^2, m_t^2)\Big\} \ . \qquad (18)$$

The cut-off independent part can also be calculated explicitly for Euclidean $\bar{q}^2 = -q^2 > 0$, and then it can be analytically continued into the physical region $q^2 > 0$. The value of this part, however, depends on the choice of the regularization (cut-off procedure). We will ignore these terms; they are smaller than the $\ln(\Lambda^2/m_t^2)$-term even in the case of low $\Lambda \sim 1$ TeV – typically by a factor of 3 or more. This approximation will not affect our results appreciably, because even the $\ln\Lambda$-terms from the above Σ_{HH}^{tt} will contribute only a relative correction of less than 10 percent to the physical parameters of our concern.

Inserting (18) into (16), we obtain the φ-renormalization parameter δZ_φ

$$\delta Z_\varphi = \kappa(\Lambda)\Big\{\ln\left[\frac{\Lambda^2}{m_t^{(0)2}(\Lambda)}\right] + \mathcal{O}(\Lambda^0)\Big\} \ . \qquad (19)$$

Inserting (18) into (15), the pole mass of the Higgs in the present framework acquires the form

$$\left(M_H^{\text{pole}}\right)^2 = \frac{d^2 V_{\text{eff}}^{(0+1\ell t)}}{d\varphi^2}\Big|_{\varphi=\langle\varphi\rangle_{1\ell t}}\Big\{1 - \kappa(\Lambda)\left[\ln\left(\frac{\Lambda^2}{(m_t^{(0)}(\Lambda))^2}\right) + \mathcal{O}(\Lambda^0)\right]\cdots\Big\} \ , \qquad (20)$$

where $(\ldots)$ represent higher powers of $\kappa\ln(\Lambda/m_t)$. These terms are small, because for $m_t \approx 180$ GeV we have $\kappa \approx 2\cdot 10^{-2}$ (– if we for a moment ignore the differences between the physical and bare quantities). The second derivative appearing in the above relation can be directly calculated from (3)-(5) and the "gap" equation (6)

$$\frac{d^2 V_{\text{eff}}^{(0+1\ell t)}}{d\varphi^2}\Big|_{\varphi=\langle\varphi\rangle_{1\ell t}} = \Big\{\frac{\lambda(\Lambda)}{3} + 2\kappa(\Lambda)g_t^2(\Lambda)\left[\ln\ \frac{\Lambda^2}{m_t^{(0)2}(\Lambda)} - 1 + \mathcal{O}\left(\frac{m_t^2}{\Lambda^2}\right)\right]\Big\}\langle\varphi\rangle_{1\ell t}^2 \ . \qquad (21)$$

The formulas (20) and (21) give us a connection between M_H^{pole} and the bare coupling $\lambda(\Lambda)$ in terms of the cut-off Λ and the bare values for the Yukawa coupling and the VEV (at the "running" energy of the upper cut-off $E \approx \Lambda$). We would like to have a corresponding relation in terms of renormalized quantities $g_t^{\text{ren.}}$ and v, because then we can find the connection between the Λ-upper bound (12) and the physical mass M_H^{pole} in terms of these well-known renormalized quantities.

Within the present framework, we have the following relation for the Yukawa coupling

$$g_t(\Lambda) = Z_{gt}^{-1/2} g_t^{\text{ren.}} \ , \qquad \left(\Rightarrow \kappa(\Lambda) = Z_{gt}^{-1}\kappa_{\text{ren.}}\right) \ , \qquad (22)$$

where the renormalization constant Z_{gt} can most easily be obtained by considering the renormalization group equation (RGE) for the g_t, running it down from $E \approx \Lambda$ to $E \approx m_t$

$$Z_{gt} = 1 + \delta Z_{gt} \ ; \qquad \delta Z_{gt} = -\frac{3}{2}\delta Z_{\varphi} + 2\delta Z_{gl} \ . \tag{23}$$

Here, we use δZ_{φ} of (19), and the gluonic renormalization effects are parametrized by δZ_{gl}

$$\delta Z_{gl} = \frac{\alpha_s}{\pi}\left[\ln\frac{\Lambda^2}{m_t^{(0)2}(\Lambda)} + \mathcal{O}(\Lambda^0)\right] \ . \tag{24}$$

We took here into account only the numerically dominant contributions to the RGE-running of g_t: the QCD contribution $2\delta Z_{gl}$ ($\propto \alpha_s/\pi$, where $\alpha_s \approx \alpha_s(E \gtrsim m_t) \approx 0.10$), and the Yukawa contribution $-3\delta Z_{\varphi}/2$ ($\propto \kappa(\Lambda)$, where $\kappa(\Lambda) \sim \kappa_{\text{ren.}} = 2.03 \cdot 10^{-2}$, for $m_t^{\text{phy}} = 180$ GeV). Combining (22)–(24) with the known effect of the "running" of the VEV between m_t and Λ (17), we also find the desired connection between the bare and the renormalized mass of the top quark

$$m_t^{(0)}(\Lambda) = \left(\frac{g_t(\Lambda)\langle\varphi\rangle_{1\ell t}}{\sqrt{2}}\right) = \left(1 - \frac{1}{2}\delta Z_{gt} - \frac{1}{2}\delta Z_{\varphi}\right) m_t^{\text{phy}} = \left(1 - \delta Z_{gl} + \frac{1}{4}\delta Z_{\varphi}\right) m_t^{\text{phy}} \ . \tag{25}$$

One may ask the question whether δZ_{gt} in (23)–(25) is really free of any Λ^2-terms, especially since the RGE for g_t would ignore any such terms. This is equivalent to the question whether the combination $(\delta Z_{gt} + \delta Z_{\varphi})$ in (25) is free of any Λ^2-terms, since δZ_{φ} was shown explicitly to satisfy this condition (cf. (16)–(19)), in the present framework where we ignore the 1-loop scalar self-interaction effects. Indeed, the combination $(\delta Z_{gt} + \delta Z_{\varphi})$ in (25) can also be obtained independently, by calculating the 1-loop contributions of the scalar and the quark sectors (taking g_t as the only nonzero Yukawa coupling) to the propagator of the top quark and finding the corresponding change of the pole mass $\delta m_t = m_t^{\text{phy}} - m_t^{(0)}(\Lambda)$. It is crucial, however, to take in the tree-level propagator the mass equal to $m_t^{(0)}(\Lambda)$ of (7). In the resulting δm_t *no* Λ^2-terms appear, i.e., the tadpole diagrams (Fig. 3) do not contribute, as ensured by the "gap" equation (6). As a matter of fact, it can be shown that the condition of the cancelation of the tadpole diagrams of Fig. 3 is equivalent to this "gap" equation. All Λ^2-terms are contained in the bare mass $m_t^{(0)}(\Lambda)$.

Now, we can express the bare quantities appearing in the preceding equations through the following renormalized known quantities

$$\begin{aligned} m_t^{\text{phy}} &= 180 \text{ GeV} \ , \qquad \langle\varphi_{\text{ren.}}\rangle = v = 246.2 \text{ GeV} \ , \\ g_t^{\text{ren.}} &= \frac{m_t^{\text{phy}}\sqrt{2}}{v} = 1.034 \ , \qquad \kappa_{\text{ren.}} = \frac{(g_t^{\text{ren.}})^2 N_{\text{c}}}{16\pi^2} = 2.03 \cdot 10^{-2} \ . \end{aligned} \tag{26}$$

For simplicity, we will omit from now on any superscripts or subscripts "phy", "pole", "ren." for the parameters m_t, g_t, κ and M_H. Unless otherwise stated, these parameters will be the physical ones. On the other hand, we continue to denote by φ the bare scalar field $\varphi^{(\Lambda)}$.

Furthermore, in the spirit of perturbation, we can replace in the relations (19)-(20) and (24) the bare quantities $\kappa(\Lambda)$ and $m_t^{(0)}(\Lambda)$ by the corresponding renormalized ones (26)

$$\delta Z_\varphi = \kappa \left[\ln \frac{\Lambda^2}{m_t^2} + \mathcal{O}\left(\Lambda^0\right) \right] \ , \quad \delta Z_{gl} = \frac{\alpha_s}{\pi} \left[\ln \frac{\Lambda^2}{m_t^2} + \mathcal{O}(\Lambda^0) \right] \ , \quad \left(\delta Z_{gt} = -\frac{3}{2} \delta Z_\varphi + 2 \delta Z_{gl} \right) . \tag{27}$$

Relations (17)-(27) allow us now to express the physical (pole) mass M_H of (20) in terms of the known renormalized parameters (26) and of the bare coupling parameter $\lambda(\Lambda)$ and the cut-off Λ

$$M_H^2 = \frac{\lambda(\Lambda) v^2}{3} \Big\{ 1 + \delta Z_\varphi \left[-2 + \frac{12 m_t^2}{\lambda(\Lambda) v^2} \right] + \mathcal{O}\left(\frac{\kappa g_t^2}{\lambda(\Lambda)} \right) \Big\} \ . \tag{28}$$

We can now finally express the upper bound (12) for the cut-off Λ in terms of the renormalized parameters M_H, m_t, κ with the help of relation (28) and "renormalization" relations (17), (22)–(23), (27)

$$\Lambda^2 \lessapprox \frac{M_H^2}{4\kappa} \frac{1}{(1-\theta(\Lambda))} \left[1 + 2\delta Z_{gl} - \delta Z_\varphi \left(\frac{1}{2} + \frac{4 m_t^2}{M_H^2} \right) + \mathcal{O}\left(\kappa, \frac{\alpha_s}{\pi} \right) \right] \ , \tag{29}$$

$$\text{when:} \qquad M^2(\Lambda) \ \geq \ 0 \ , \quad \text{or} \quad -M^2(\Lambda) = |M^2(\Lambda)| \ \ll \ \frac{\lambda(\Lambda) \langle \varphi \rangle_{1\ell t}^2}{6} \ \left(\approx \frac{\lambda(\Lambda) v^2}{6} \right) \ . \tag{30}$$

Here, $\theta(\Lambda)$ is the small parameter defined through (13) and (7). For example, for $\Lambda \sim 1$ TeV we have $\theta(\Lambda) \approx 0.1$. Within the framework of this section, the bare mass $m_t^{(0)}(\Lambda)$ is close to the physical mass $m_t = 180$ GeV. Therefore

$$\theta(\Lambda) = \frac{m_t^2}{\Lambda^2} \ln \frac{\Lambda^2}{m_t^2} \left[1 + \mathcal{O}\left(\delta Z_{gl,\varphi} \right) \right] \ . \tag{31}$$

Relation (29) gives us, after one or two iterations, an upper bound on the cut-off Λ as a function of the (physical) Higgs mass M_H^2. This upper bound arised as a consequence of the (non-perturbative) effect of the heavy top quark on the scalar sector. Alternatively, we can simply express this upper bound with the bare coupling $\lambda(\Lambda)$ instead of M_H^2, using (12) and the renormalization conditions (17) and (22)–(23)

$$\Lambda^2 \lessapprox \frac{\lambda(\Lambda) v^2}{12\kappa} \frac{1}{(1-\theta(\Lambda))} \left[1 + \delta Z_{gt} - \delta Z_\varphi \right] \ , \tag{32}$$

where the conditions for the inequality are those in (30). We note that for $|M(\Lambda)| \ll v\sqrt{\lambda(\Lambda)/6}$ ($\approx M_H/\sqrt{2}$), the actual ultraviolet cut-off Lambda becomes approximately equal to the upper bound in (29), (32) .

Some numerical results of this relation ($\lambda(\Lambda)$ vs. M_H vs. $\Lambda^{\text{u.b.}}$) are given in the first three columns of Table 1. We note that we obtain rather low upper bounds $\Lambda^{\text{u.b.}} \lessapprox 1$ TeV.

These results basically survive even in the case when $-M^2(\Lambda) = |M^2(\Lambda)| = \mathcal{O}(\lambda v^2)$ ($= \mathcal{O}(M_H^2)$ by (28)). The values of M_H as a function of $\lambda(\Lambda)$ and Λ remain unaffected then, as seen by formula (28). This is so because any $M^2(\Lambda)$-dependence there has been eliminated by the use of the "gap" equation (6). The expression on the r.h.s. of (32), however, is modified in such a case by the replacement: $\lambda(\Lambda)\langle\varphi\rangle^2_{1\ell t}/6 \mapsto m^2(\Lambda)$, as seen from (12) and (9). This result in an additional factor $k(\Lambda)$ of order 1 on the r.h.s. of (32)

$$\Lambda^2 = \frac{k(\Lambda)\lambda(\Lambda)v^2}{12\kappa} \frac{1}{(1-\theta(\Lambda))} \left[1 + \delta Z_{gt} - \delta Z_{\varphi}\right] \ , \tag{33}$$

$$\text{where:} \quad k(\Lambda) = \left[1 + \frac{6|M^2(\Lambda)|}{\lambda(\Lambda)v^2}(1+\delta Z_{\varphi})\right] \ , \qquad \text{for:} \quad -M^2(\Lambda) = |M^2(\Lambda)| = \mathcal{O}(\lambda(\Lambda)v^2) \ . \tag{34}$$

Therefore, $\Lambda = \mathcal{O}(1$ TeV) also in this case.

We stress that all these results are in the framework of the present Section where the 1-loop contributions of the scalar self-interaction and of the electroweak gauge bosons to V_{eff} have been ignored. In the next Section, we will include these contributions.

3 Inclusion of 1-loop scalar and gauge bosonic contributions

It is straightforward to obtain from (3) and (5) an explicit expression for the effective potential with the leading heavy top quark contributions included

$$V_{\text{eff}}^{(0+1\ell t)}\left(\varphi^2;\Lambda\right) = -\frac{1}{2}M_0^2\varphi^2 + \frac{1}{4!}\lambda_0\varphi^4 - \frac{1}{4}\kappa(\Lambda)g_t^2(\Lambda)\varphi^4 \ln\frac{\varphi^2}{\langle\varphi\rangle^2_{1\ell t}} + \mathcal{O}\left(\Lambda^{-2}\right) \ , \tag{35}$$

where we denoted

$$\begin{aligned} M_0^2 &= M^2(\Lambda) + 2\kappa(\Lambda)\Lambda^2 = \left[\frac{\lambda(\Lambda)}{6} + g_t^2\delta Z_{\varphi}\left(1 + \mathcal{O}\left(1/\ln\frac{\Lambda^2}{m_t^2}\right)\right)\right]\langle\varphi\rangle^2_{1\ell t} \ , \\ \lambda_0 &= \lambda(\Lambda) + 6g_t^2\delta Z_{\varphi}\left(1 + \mathcal{O}\left(1/\ln\frac{\Lambda^2}{m_t^2}\right)\right) \ . \end{aligned} \tag{36}$$

Here, δZ_{φ} is the expression written in (19), or in an approximate form in (27). Strictly speaking, the Yukawa coupling in (36) is the bare one ($g_t(\Lambda)$), and δZ_{φ} is defined by (16) (with $\Sigma_{HH} \mapsto \Sigma^{tt}_{HH}$) and (19). It contains the "bare" mass $m_t^{(0)}(\Lambda)$ defined in (7). However, since it will turn out that $\langle\varphi\rangle^2 \sim \langle\varphi\rangle^2_{1\ell t}$, and since we will neglect the terms $\mathcal{O}(1/\ln(\Lambda^2/m_t^2))$ in (36), we can use instead there the δZ_{φ} given by (27) in terms of physical quantities. The errors due to the replacement $g_t(\Lambda) \mapsto g_t$

in (36) are even smaller (cf. (22)–(24) and (27)). The expression on the r.h.s. of the first line in (36) was obtained from the "gap" equation (6).

The effective potential (35) is now used as the starting point (i.e., "tree level") to calculate the perturbative 1-loop corrections to it coming from the scalar sector itself and from the sector of the electroweak gauge bosons. This can be done in a straightforward way by using, for example, the standard path integral approach [6]. This leads us to the following corrections to $V_{\rm eff}$:

$$\begin{aligned}
\delta V_{\rm eff}^{(1\ell sc)}\left(\varphi^2;\Lambda\right) \quad = \quad & \frac{1}{64\pi^2}\Big\{ 2\Lambda^2 \left[\left(\frac{\lambda_0}{2}\varphi^2 - M_0^2\right) + 3\left(\frac{\lambda_0}{6}\varphi^2 - M_0^2\right) + \mathcal{O}\left(\kappa g_t^2 \varphi^2\right)\right] \\
& - \left(\ln\frac{\Lambda^2}{m_t^2}\right)\left[\left(\frac{\lambda_0}{2}\varphi^2 - M_0^2 + \mathcal{O}\left(\kappa g_t^2\varphi^2\right)\right)^2 + 3\left(\frac{\lambda_0}{6}\varphi^2 - M_0^2 + \mathcal{O}\left(\kappa g_t^2\varphi^2\right)\right)^2\right] \\
& + \mathcal{O}\left((\lambda_0\varphi^2/3)^2\right)\Big\} \ , \qquad (37)
\end{aligned}$$

$$\delta V_{\rm eff}^{(1\ell gb)}\left(\varphi^2;\Lambda\right) = \frac{3}{32\pi^2}\Big\{ \int_0^{\Lambda^2} d\bar{q}^2 \bar{q}^2 \ln\left[\frac{\bar{q}^2 + M_Z^2\varphi^2/v^2}{\bar{q}^2}\right] + 2\int_0^{\Lambda^2} d\bar{q}^2\bar{q}^2 \ln\left[\frac{\bar{q}^2 + M_W^2\varphi^2/v^2}{\bar{q}^2}\right]\Big\} \ . \quad (38)$$

The latter expression was calculated in the Landau gauge ($\xi \to \infty$), since only in this gauge the coupling of ghosts to scalars is zero and we don't have any 1-loop contribution of ghosts to $V_{\rm eff}$. Furthermore, we replaced in (38) the combinations $g^2(\Lambda) + g'^2(\Lambda)$ and $g^2(\Lambda)$ of the bare electroweak coupling parameters by their tree level approximations $4M_Z^2/v^2$ and $4M_W^2/2$, respectively. The error arising from this replacement will be very minor (for $\Lambda = \mathcal{O}(1 \text{ TeV})$).

We note that the expression (37), representing the 1-loop contribution of the Higgs and of the three Goldstones to the effective potential $V_{\rm eff}$, should be regarded as a *perturbative* correction to $V_{\rm eff}^{(0+1\ell t)}$. This is so because (37) represents an effect of the scalar sector on *itself*, and the higher loop contributions of such effects would be proportional to the correspondingly higher powers in the modified scalar self-interaction parameter λ_0. It would be inconsistent to equate in such a series the terms corresponding to different powers of λ_0. Hence, we are able to make reasonable predictions about these corrections only as long as the scalar sector is not too strongly self-interacting, i.e., as long as these corrections do not drastically change, through its contributions, the VEV $\langle\varphi\rangle_{1\ell t}^2$ and the upper bound $(\Lambda_{1\ell t}^{\rm u.b.})^2$ of the previous Section – say, by not more than 50 percent. We will see soon that this will restrict our predictions to a region of Higgs masses $M_H \stackrel{<}{\approx} 250$ GeV.

On the other hand, the 1-loop corrections (38) to $V_{\rm eff}$ from the sector of the electroweak gauge bosons can, in principle, be regarded as valid also in the non-perturbative region – i.e., if the gauge boson masses and the resulting corrections to the "gap" equation were large. However, for the experimentally well-known values of M_Z and M_W, these contributions will turn out to be quite small, and we will treat them as perturbative corrections.

The "gap" equation (6), which determines the bare VEV when the corrections (37) and (38) to $V_{\rm eff}$ are ignored, is now modified perturbatively

$$\frac{d}{d\varphi^2}\left(V_{\rm eff}^{(0+1\ell t)}+\delta V_{\rm eff}^{(1\ell sc)}+\delta V_{\rm eff}^{(1\ell gb)}\right)\Big|_{\varphi=\langle\varphi\rangle}=0 \qquad \Longrightarrow$$

$$\delta\langle\varphi\rangle^2\frac{d^2V_{\rm eff}^{(0+1\ell t)}}{d\left(\varphi^2\right)^2}\Bigg|_{\varphi^2=\langle\varphi\rangle^2_{1\ell t}}+\frac{d\left(\delta V_{\rm eff}^{(1\ell sc)}\right)}{d\varphi^2}\Bigg|_{\varphi^2=\langle\varphi\rangle^2_{1\ell t}}+\frac{d\left(\delta V_{\rm eff}^{(1\ell gb)}\right)}{d\varphi^2}\Bigg|_{\varphi^2\approx v^2}\approx 0\ , \tag{39}$$

where we denoted

$$\delta\langle\varphi\rangle^2=\langle\varphi\rangle^2-\langle\varphi\rangle^2_{1\ell t}\ . \tag{40}$$

We mean by $\langle\varphi\rangle$ the corrected VEV of the (bare) scalar field $\varphi=\varphi^{(\Lambda)}$. In the perturbative approach, we demand that the correction (40) be relatively small. That's why we included in the corrected "gap" equation (39) only the first two terms in the Taylor expansion of the derivative $dV_{\rm eff}^{(0+1\ell t)}/d\varphi^2$ around the value $\varphi^2=\langle\varphi\rangle^2_{1\ell t}$. Note that the latter value, by definition, satisfies the (0+1-loop top) "gap" equation (6). On the other hand, the derivative of the (smaller) corrective potentials $\delta V_{\rm eff}^{(1\ell sc)}$ and $\delta V_{\rm eff}^{(1\ell gb)}$ in (39) are evaluated, according to convenience, at one of the following similar values of VEV: $\langle\varphi\rangle^2_{1\ell t}$ and $\langle\varphi_{\rm ren.}\rangle^2=v^2$, i.e., here we take just the first term in Taylor expansions.

Explicit calculation of (39) then leads to the following expression for the VEV correction $\delta\langle\varphi\rangle^2$:

$$\begin{aligned}\delta\langle\varphi\rangle^2 \;&=\; -\frac{3}{8\pi^2}\Lambda^2\left[1+\mathcal{O}\left(\frac{\kappa g_t^2}{\lambda_0}\right)+\mathcal{O}\left(\theta(\Lambda)\right)\right]+\\ &\quad -\frac{9}{8\pi^2}\frac{1}{\lambda_0}\Lambda^2\left(\frac{M_Z^2+2M_W^2}{v^2}\right)\left[1+\mathcal{O}\left(\frac{M_Z^2}{\Lambda^2}\ln\frac{\Lambda^2}{M_Z^2}\right)+\mathcal{O}\left(\frac{\kappa g_t^2}{\lambda_0}\right)\right]\ .\end{aligned} \tag{41}$$

We note that the (0+1-loop top) "gap" equation (6) defines $\langle\varphi\rangle^2_{1\ell t}$ in the first place. From there we obtained, for the case of $M^2(\Lambda)\geq 0$ or $-M^2(\Lambda)=|M^2(\Lambda)|\ll\lambda(\Lambda)\langle\varphi\rangle^2_{1\ell t}/6$, the upper bound (12) for Λ, expressed in terms of the bare VEV $\langle\varphi\rangle^2_{1\ell t}$. This upper bound is, of course, still valid in our case. However, the bare VEV $\langle\varphi\rangle^2_{1\ell t}$ in (12) cannot be regarded in the framework of the present Section as the actual bare VEV. We express $\langle\varphi\rangle^2_{1\ell t}$ in (12) as the difference between the actual (i.e., corrected) bare VEV $\langle\varphi\rangle^2$ and the scalar and gauge bosonic corrections $\delta\langle\varphi\rangle^2$ of (41). Thus we obtain, for the case of $M^2(\Lambda)\geq 0$ or $-M^2(\Lambda)=|M^2(\Lambda)|\ll\lambda(\Lambda)v^2$, the following upper bound for the ultraviolet cut-off Λ, now modified in comparison to (32) perturbatively by the 1-loop scalar and 1-loop gauge bosonic contributions

$$\begin{aligned}\Lambda^2 \;&\lessapprox\; \frac{\lambda(\Lambda)v^2}{12\kappa}\frac{1}{(1-\theta(\Lambda))}\left[1+\delta Z'_{gt}-\delta Z'_{\varphi}\right]\Bigg\{1+\frac{\lambda(\Lambda)}{32\pi^2\kappa}\left[1+\mathcal{O}\left(\frac{\kappa g_t^2}{\lambda(\Lambda)}\right)+\mathcal{O}\left(\theta(\Lambda)\right)+\mathcal{O}\left(\delta Z_{gt,\varphi}\right)\right]\\ &\quad+\frac{3}{32\pi^2\kappa}\frac{\lambda(\Lambda)}{\lambda_0}\left(\frac{M_Z^2+2M_W^2}{v^2}\right)\left[1+\mathcal{O}\left(\frac{M_Z^2}{\Lambda^2}\ln\frac{\Lambda^2}{M_Z^2}\right)+\mathcal{O}\left(\frac{\kappa g_t^2}{\lambda(\Lambda)}\right)++\mathcal{O}\left(\delta Z_{gt,\varphi}\right)\right]\Bigg\}\ ,\end{aligned} \tag{42}$$

The parameter λ_0 appearing above was defined in (36) in terms of the bare parameter $\lambda(\Lambda)$ and the cut-off Λ. The parameters $\delta Z'_\varphi$ and $\delta Z'_{gt}$ are perturbative renormalization effects for g_t and φ

$$\langle\varphi\rangle^2 = \left(1 - \delta Z'_\varphi\right) v^2 \ , \quad g_t^2(\Lambda) = \left(1 - \delta Z'_{gt}\right) g_t^2 \ , \quad \left(\Rightarrow \kappa(\Lambda) = (1 - \delta Z'_{gt}))\kappa \ ,\right) \ . \tag{43}$$

They are similar to the expressions δZ_φ and δZ_{gt} of the previous Section (cf. eqs. (19) and (23)–(24)), the only difference being the modifications by contributions of the electroweak gauge bosons (see eqs. (46) and (47) and the discussion below). The small parameter $\theta(\Lambda)$ appearing in (42) was defined through (13) and (7). Therefore, it can be written now in terms of Λ and the physical parameter m_t

$$\theta(\Lambda) = \frac{m_t^2}{\Lambda^2}\left[1 + \frac{|\delta\langle\varphi\rangle^2|}{v^2}\right] \ln\left[\frac{\Lambda^2}{m_t^2\left(1 + |\delta\langle\varphi\rangle^2|/v^2\right)}\right]\left[1 + \mathcal{O}\left(\delta Z'_{\varphi,gt}\right)\right] \ , \tag{44}$$

where $\delta\langle\varphi\rangle^2 = -|\delta\langle\varphi\rangle^2|$ as a function of Λ is given in (41). Note that $|\delta\langle\varphi\rangle^2|/v^2 \lesssim 1$, because we restrict ourselves to perturbative effects of the scalar and gauge bosonic sector. All the other parameters appearing in (42) are well known (cf. (26), and: $M_W \approx 80.2$ GeV, $M_Z \approx 91.2$ GeV). Relation (42) is now the modified version of relation (32), expressing the upper bound of the ultraviolet cut-off with the bare self-coupling parameter $\lambda(\Lambda)$. In this relation, the second term in the curly brackets represents the modification due to the 1-loop scalar self-interaction contributions, and the third the modification due to the 1-loop gauge bosonic contributions to the effective potential. Strictly speaking, we obtain an infinite geometric series of such terms; however, in the spirit of perturbation, we ignore all the terms of higher powers in $\lambda(\Lambda)/(32\pi^2\kappa)$ and $M_{g.b.}^2/(32\pi^2\kappa v^2)$.

As already indicated in the previous paragraph, the inclusion of the 1-loop scalar and gauge bosonic contributions to the "gap" equation requires, for reasons of consistency, that we also modify the parameter δZ_φ of eqs. (16) and (19): $\delta Z_\varphi \mapsto \delta Z'_\varphi$. The modification is caused by the 1-loop contributions of scalars and gauge bosons to the derivative $d\Sigma_{HH}(q^2)/dq^2$. It turns out that the scalar self-interactions alone do not contribute to this derivative because they give q^2-independent contribution to the Λ-dependent part of Σ_{HH}. The electroweak gauge bosons and Goldstones do contribute. The contributing diagrams are depicted in Fig. 2b. The contribution Σ_{HH}^{tt} of the diagram of Fig. 2a has already been calculated in the previous Section (14)–(18). In a completely analogous way, we obtain for the truncated Green functions of Fig. 2b [6]

$$\Sigma_{HH}^{ZG}(q^2) + \Sigma_{HH}^{WG}(q^2) = \frac{3}{16\pi^2\langle\varphi\rangle^2} q^2\left(M_Z^2 + 2M_W^2\right)\left[\ln\left(\frac{\Lambda^2}{m_t^2}\right) + \mathcal{O}\left(\Lambda^0\right)\right] \ . \tag{45}$$

[6] Calculated in the Landau gauge, for reasons explained in the text following eq. (38).

Since $\delta Z'_\varphi = -d\Sigma_{HH}/dq^2|_{q^2=M_H^2}$ (cf. eq. (16)), this leads to the following modification

$$\Sigma_{HH} \mapsto \Sigma_{HH}^{tt} + \left(\Sigma_{HH}^{ZG} + \Sigma_{HH}^{WG}\right) \qquad \Rightarrow \qquad \delta Z_\varphi \mapsto \delta Z'_\varphi = \kappa' \left[\ln\left(\frac{\Lambda^2}{m_t^2}\right) + \mathcal{O}\left(\Lambda^0\right)\right] ,$$

$$\text{where:} \qquad \kappa' = \kappa - \frac{3}{16\pi^2}\frac{(M_Z^2 + 2M_W^2)}{v^2} = 1.37 \cdot 10^{-2} . \tag{46}$$

We note that we can obtain this result also very quickly by looking at the 1-loop renormalization group equation (RGE) for the "running" VEV $v(E) = \langle \varphi^{(E)} \rangle$. Such an RGE can be found, for example, in ref. [7].

Furthermore, for the sake of consistency, we also include the 1-loop contributions of the electroweak gauge bosons to the renormalization of the Yukawa coupling parameter g_t (cf. (22)–(24)). They can most easily be obtained by from the 1-loop RGE [7] for the Yukawa coupling g_t

$$Z_{gt} \mapsto Z'_{gt} = +2\delta Z_{gl} - \frac{3}{2}\delta Z_\varphi + \frac{1}{48\pi^2}\frac{(17M_Z^2 + 10M_W^2)}{v^2}\left[\ln\left(\frac{\Lambda^2}{m_t^2}\right) + \mathcal{O}\left(\Lambda^0\right)\right] , \tag{47}$$

where δZ_{gl} and δZ_φ are given in (27). This could be obtained also in a more tedious way, by looking at the 1-loop contributions of the electroweak gauge bosons and Goldstones to the top quark propagator and finding the corresponding change of the pole mass (taking only the Λ-dependent part). This change would correspond to the gauge bosonic and Goldstone contribution in the combination $(\delta Z'_\varphi + \delta Z'_{gt})/2$, since $m_t \propto \langle \varphi \rangle g_t$. One may ask whether this combination, or equivalently $\delta Z'_{gt}$, is free of Λ^2-terms (which are ignored by RGEs). The answer is yes, because such terms come from tadpole diagrams and they cancel out due to the "gap" equation (39). The argument is closely analogous to that presented in the previous Section after eq. (25). All Λ^2-terms in the physical (pole) mass m_t are already contained in the bare mass factor $g_t(\Lambda)\langle\varphi\rangle/\sqrt{2}$, i.e., in the VEV $\langle\varphi\rangle$ of (39), and none are in the radiative correction factor $1 + (\delta Z'_\varphi + \delta Z'_{gt})/2$ (cf. (25)).

In order to complete the Section, we must also find the physical (pole) mass M_H of the Higgs in terms of Λ and the bare coupling $\lambda(\Lambda)$. Now, M_H must be corrected by the 1-loop scalar and electroweak gauge bosonic effects. The formula (15) still applies, except that now we have to calculate the second derivative of the effective potential modified by the scalar and gauge bosonic contributions (37) and (38), and the truncated Green function Σ_{HH}^{tt} should be replaced by the full 1-loop truncated Green function Σ_{HH} with the gauge bosonic and scalar loop contributions included

$$M_H^2 = \frac{d^2}{d\varphi^2}\left(V_{\text{eff}}^{(0+1\ell t)} + \delta V_{\text{eff}}^{(1\ell sc)} + \delta V_{\text{eff}}^{(1\ell gb)}\right)\Big|_{\varphi=\langle\varphi\rangle} + \left[\Sigma_{HH}\left(q^2 = M_H^2\right) - \Sigma_{HH}\left(q^2 = 0\right)\right] . \tag{48}$$

[7] From this RGE we see that, at 1-loop, there are no scalar self-interaction contributions to the running of g_t.

It is straightforward to check that, again, only the diagrams of Figs. 2a-2b contribute to the difference of Σ's in (48), i.e., Σ_{HH} in (48) can be taken to be the sum of the epressions (18) (with $m_t^{(0)}(\Lambda)$ there replaced by m_t) and (45). Furthermore, the second derivative in (48) can be calculated again in the spirit of perturbation

$$\frac{d^2}{d\varphi^2}\left(V_{\rm eff}^{(0+1\ell t)} + \delta V_{\rm eff}^{(1\ell sc)} + \delta V_{\rm eff}^{(1\ell gb)}\right)\Big|_{\varphi=\langle\varphi\rangle} \approx$$

$$\approx \left[\frac{d^2 V_{\rm eff}^{(0+1\ell t)}}{d\varphi^2} + \delta\langle\varphi\rangle \frac{d^3 V_{\rm eff}^{(0+1\ell t)}}{d\varphi^3}\right]\Bigg|_{\varphi=\langle\varphi\rangle_{1\ell t}} + \frac{d^2\left(\delta V_{\rm eff}^{(1\ell sc)}\right)}{d\varphi^2}\Bigg|_{\varphi=\langle\varphi\rangle_{1\ell t}} + \frac{d^2\left(\delta V_{\rm eff}^{(1\ell gb)}\right)}{d\varphi^2}\Bigg|_{\varphi\approx v} . \qquad (49)$$

Similarly as in (39), we used here only the first two terms in the Taylor expansion of the second derivative $d^2 V_{\rm eff}^{(0+1\ell t)}/d\varphi^2$ around the value $\varphi = \langle\varphi\rangle_{1\ell t}$. The second derivatives of the corrective potentials $\delta V_{\rm eff}^{(1\ell sc)}$ and $\delta V_{\rm eff}^{(1\ell gb)}$ are evaluated, according to convenience, at one of the following similar values: $\langle\varphi\rangle_{1\ell t}$ and $\langle\varphi_{\rm ren.}\rangle = v$. We replace $\delta\langle\varphi\rangle$ by $\delta\langle\varphi\rangle^2/(2\langle\varphi\rangle_{1\ell t})$, and use relation (41) for $\delta\langle\varphi\rangle^2$. We then directly calculate (49) in this way and with help of the results obtained so far. Then we insert the obtained expression into relation (48) and end up with the following square of the physical mass of the Higgs

$$\begin{aligned} M_H^2 &= \frac{\lambda(\Lambda) v^2}{3}\left[1 - 2\delta Z'_\varphi + \delta Z_\varphi \frac{3\cdot 4 m_t^2}{\lambda(\Lambda) v^2} + \mathcal{O}\left(\frac{\kappa g_t^2}{\lambda(\Lambda)}\right)\right] \\ &\quad - \frac{3v^2}{8\pi^2}\left[\frac{\lambda^2(\Lambda)}{9}\ln\frac{\Lambda^2}{m_t^2} + \frac{M_Z^4}{v^4}\ln\frac{\Lambda^2}{M_Z^2} + 2\frac{M_W^4}{v^4}\ln\frac{\Lambda^2}{M_W^2}\right]\left[1 + \mathcal{O}\left(\delta Z_\varphi\right) + \mathcal{O}\left(1/\ln\frac{\Lambda^2}{m_t^2}\right)\right] \\ &\quad + \mathcal{O}\left(\frac{\kappa g_t^2 \Lambda^2}{16\pi^2}\right) . \end{aligned} \qquad (50)$$

The leading part of the Λ^2-dependence in M_H^2, i.e., the terms $\mathcal{O}\left(\Lambda^2\lambda(\Lambda)/(16\pi^2)\right)$ and $\mathcal{O}\left(\Lambda^2 M_{g.b.}^2/(16\pi^2 v^2)\right)$, turn out to have the coefficient equal to zero. The remaining Λ^2-terms are suppressed by a factor which is at most of order $(\kappa g_t^2)/\pi^2$, as indicated in the last line of (50). These terms may appear and would have its origin partly in the (neglected) terms $\Lambda^2\mathcal{O}(\kappa g_t^2\varphi^2)/(32\pi^2)$ of the scalar-induced effective potential $\delta V_{\rm eff}^{(1\ell sc)}$ of (37). For $\Lambda \lessapprox 1.5$ TeV, the terms $\mathcal{O}\left(\kappa g_t^2\Lambda^2/\pi^2\right)$ appear not to surpass the $\ln\Lambda$-terms of the second line in (50) and appear to change the value of M_H^2 in such a case at most by ten percent. A more detailed analysis should also include these possible terms in M_H^2. Here we will neglect them and consider the formula (50) only as an approximation that results in an estimated overall error of ten percent or less for M_H, in the case of relatively low ultraviolet cut-offs $\Lambda \leq 1.5$ TeV of Table 1. At the end of this Section we will discuss other contributions to the estimated overall error of formula (50).

At this point, we are able to calculate the ultraviolet upper bounds $\Lambda^{\rm u.b.}$ from (42), and the (physical) Higgs masses M_H from (50) – both interrelated as functions of one single unknown variable,

the bare scalar self-coupling parameter $\lambda(\Lambda)$. The results are presented in the last two columns of Table 1. For comparison, we also included the values when the gauge bosonic 1-loop effects were neglected (fourth and fifth column). The values from Section 1, when the 1-loop scalar self-interaction effects were neglected as well, are in the second and third columns.

As emphasized at the beginning of the present Section, we confined ourselves only to such values of $\lambda(\Lambda)$ for which the calculated values have predictive power – i.e., we confined ourselves to the cases when the 1-loop scalar self-interaction effects on the effective potential remain perturbative, changing (increasing) the square of the predicted ultraviolet cut-off Λ by roughly 50 percent or less. On the other hand, it can be seen, by using (41), that the resulting values for Λ (Table 1, last column) change (decrease) the square of the VEV typically by 50 percent or less: [8] $|\delta\langle\varphi\rangle^2| = \langle\varphi\rangle^2_{1\ell t} - \langle\varphi\rangle^2 \stackrel{<}{\approx} 0.5\langle\varphi\rangle^2_{1\ell t}$. This means that the effects of $\delta V_{\text{eff}}^{(1\ell sc)}$ and $\delta V_{\text{eff}}^{(1\ell gb)}$ do not wash out the VEV $\langle\varphi\rangle_{1\ell t}$ to the value zero or almost zero. If they did, they would make our assumption of the perturbative nature of these effects non-viable in retrospect.

The philosophy followed in the present paper is essentially different from that of the authors of [8]-[10]. They assumed that a new physics (at $E \stackrel{>}{\approx} \Lambda$) protects with a symmetry the masses of the scalars and of the top quark from acquiring Λ^2-dependent (and even $\ln\Lambda$-dependent, cf. [9]-[10]) terms. In such a case, the tree level VEV ($\langle\varphi\rangle_0 = \sqrt{6M^2(\Lambda^2)/\lambda(\Lambda)}$) would still be drastically changed by the separate quantum contributions of the top quark, the scalar and the electroweak gauge boson sectors. However, these potentially huge contributions would largely cancel each other, thus resulting in the possibility of having very high cut-off Λ, even in the case of the tree-level SSB. If using the effective potential of the present paper, the cancelation relations of [8]-[10] can be reproduced by requiring that the Λ^2 and $\ln\Lambda$-terms appearing in the first derivative $dV_{\text{eff}}/d\varphi^2|_{\varphi=\langle\varphi\rangle}$ of the entire (0+1)-loop effective potential ($V_{\text{eff}}^{(0+1\ell t)} + \delta V_{\text{eff}}^{(1\ell sc)} + \delta V_{\text{eff}}^{(1\ell gb)}$) are zero. In the terminology of the present paper, we would have in such a case essentially the cancelation of the leading top quark and scalar self-interaction quantum effects: $\langle\varphi\rangle^2_{1\ell t} + \delta\langle\varphi\rangle^2 \approx \langle\varphi\rangle^2_0$. This would require the scalar self-interactions to be stronger than those assumed in the present paper, and therefore the Higgs mass would be heavier ($M_H \stackrel{>}{\approx} 300$ GeV, cf. [10]) than the highest Higgs mass of Table 1 ($M_H \approx 235$ GeV).

We stress that the results of Table 1 are valid only for the case when the bare parameters $M^2(\Lambda)$ and $\lambda(\Lambda)$ of the starting tree level scalar potential (3) satisfy either one of the two conditions (30)

[8] In the extreme case of $\lambda(\Lambda) = 3.00$ in Table 1, the square of the new VEV $\langle\varphi\rangle^2$ is decreased to 42 percent of the value of $\langle\varphi\rangle^2_{1\ell t}$, i.e., $\langle\varphi\rangle \approx 0.65\langle\varphi\rangle_{1\ell t}$ in this case.

which can be rewritten in the framework of this Section as:

$$\text{either:} \quad 0 \le M^2(\Lambda), \qquad \text{or:} \quad -M^2(\Lambda) = |M^2(\Lambda)| \ll \frac{\lambda(\Lambda)v^2}{6}\left[1+\frac{|\delta\langle\varphi\rangle^2|}{v^2}\right] \left(= \mathcal{O}(M_H^2)\right) . \quad (51)$$

Here, $\delta\langle\varphi\rangle^2 = -|\delta\langle\varphi\rangle^2|$ is given in (41) (note that $|\delta\langle\varphi\rangle^2|/v^2 \lessapprox 1$ in the discussed cases). Incidentally, this condition includes also all cases of the (tree level) SSB in (3), i.e., $M^2(\Lambda) > 0$.

However, we point out that, similarly as argued at the end of the previous Section, the results of the present Section basically survive even in the case when

$$-M^2(\Lambda) = |M^2(\Lambda)| = \mathcal{O}(\lambda v^2) \left(= \mathcal{O}(M_H^2)\right) . \quad (52)$$

The values of M_H^2 in (50) do not have an explicit dependence on $M^2(\Lambda)$ and therefore remain unchanged in such a case. The values of Λ^2, however, are then modified: they are then equal to the r.h.s. of (42) amplified by the following factor $k_*(\Lambda)$

$$k_*(\Lambda) = \left[1 + \frac{6|M^2(\Lambda)|}{\lambda(\Lambda)v^2}\left(1+\delta Z'_\varphi\right)\right] . \quad (53)$$

This factor is then of order one. Hence, we have $\Lambda = \mathcal{O}(1 \text{ TeV})$ also in the case of (52), although the numerical values of Λ's of Table 1 are not valid then.

If, on the other hand, $M^2(\Lambda)$ is negative *and* its absolute value is larger than $\lambda(\Lambda)v^2$ by at least an order of magnitude, then we have a possibility to avoid the stringent upper bounds of Table 1 for the ultraviolet cut-off Λ. In such a case, as seen already from the "gap" equation (6), we must have very large bare parameter $|M^2(\Lambda)| \approx \kappa\Lambda^2 \sim 10^{-2}\Lambda^2$, while the dimensionless bare coupling $\lambda(\Lambda)$ remains small ($\lambda(\Lambda) \sim M_H^2/v^2$). A special case of this, namely the case when the scalar doublet is not self-interacting at the cut-off scale Λ and has a finite bare mass there ($\lambda(\Lambda) \approx 0$ and $\mu^2(\Lambda) = -M^2(\Lambda) > 0$) was discussed in ref. [4]. The authors of ref. [4] obtained in such a case a wide range of possibilities for the values of Λ ($1 \text{ TeV} \lessapprox \Lambda \lessapprox E_{\text{GUT}}$). They obtained $\Lambda \approx 1$ TeV (and the physical mass $M_H \approx 80$ GeV) only if, in addition, they demanded that the fourth derivative of $V^{(0+1\ell t)}$ be zero at the VEV. The latter requirement gives in the case of non-zero (positive) small $\lambda(\Lambda)$ a higher Higgs mass (cf. (21) and (28)), but an even lower cut-off because

$$\frac{d^4 V_{\text{eff}}^{(0+1\ell t)}}{d\varphi^4}\Big|_{\varphi=\langle\varphi\rangle_{1\ell t}} = \lambda(\Lambda) + 6\kappa g_t^2\left[\ln\frac{\Lambda^2}{m_t^2} - \frac{11}{3} + \mathcal{O}\left(\frac{m_t^2}{\Lambda^2}\right)\right] . \quad (54)$$

At the end, we discuss the errors involved in the upper bounds $\Lambda^{\text{u.b.}}$ of Table 1 (seventh column). The errors appear mostly due to our neglecting those terms on the r.h.s. of (42) which we denoted

there with $\mathcal{O}(\cdots)$, and because we neglected possible terms of $\mathcal{O}(\Lambda^0)$ in the $\ln\Lambda$-dominated expressions for δZ_{gl} and δZ_{φ}. The errors for the upper bound of Λ^2 resulting from the latter approximation are estimated to be at most 9-10 percent (we take: $\mathcal{O}(\Lambda^0) = 1$ for estimate). The bulk of this error (up to 7 percent) comes from the uncertain $\mathcal{O}(\Lambda^0)$-term of the gluonic (QCD) renormalization effect δZ_{gl} (27). The terms denoted as $\mathcal{O}(\kappa g_t^2/\lambda(\Lambda))$ in (42) are magnified by a factor of order 10, as explicit preliminary calculations of these terms indicate. They then result in an error of up to 5 percent for the upper bound of Λ^2. The terms $\mathcal{O}(\theta(\Lambda))$ in (42) also result in an error of up to 5 percent. The errors from the uncertainties of the last term in the curly brackets of (42) (i.e., of the gauge bosonic contributions) are quite negligible. Finally, the perturbative approximation of (39), where we included only the leading term in the expansion of $d(\delta V_{\rm eff}^{(1\ell sc)})/\delta\varphi^2$ around the value of $\varphi^2 = \langle\varphi\rangle_{1\ell t}^2$, would also amount to a certain error in $\delta\langle\varphi\rangle^2$, and therefore in the upper bound for Λ^2. Explicit preliminary calculations, where we include also the next term in the Taylor expansion, indicate that the error committed in $\delta\langle\varphi\rangle^2$ is about 15-20 percent for the values of Λ of Table 1. This in turn would result in an error for the upper bound on Λ^2 of 8-12 percent.

Therefore, regarding all the sources of the discussed errors as being independent, we end up with a rough estimate of 10-18 percent for the possible error in the upper bound of Λ^2. This implies an estimated error of 5-10 percent for the values of the upper bound of Λ in the last column of Table 1.

For large values of $\lambda(\Lambda)$ ($\lambda(\Lambda) > 2$), we have, in addition, uncertainties arising from the "higher order" contributions $\propto \lambda^3(\Lambda)$, i.e., contributions that include also the 2-loop effects of the scalar self-interactions. They result in relative corrections to the upper bound of Λ^2 which we conjecture to be roughly of the order of $[\lambda(\Lambda)/(32\pi^2\kappa)]^2$, i.e., of the order of the next (deleted) term of the geometric series in (42). Hence, these corrections are appreciable in the cases of the upper half of the values of $\lambda(\Lambda)$ taken in Table 1. For example, for $\lambda(\Lambda) = 2.25$, 3.00, these corrections to the upper bound of Λ^2 would be roughly 12 and 22 percent, respectively; such effects would then increase the previously estimated uncertainty of 5-10 percent for $\Lambda^{\rm u.b.}$ to 8-12 and 12-15 percent, respectively. The analogous uncertainties of the gauge bosonic effects are negligible.

We wish to point out, however, that the estimated errors for the cut-off Λ should be looked upon with some reservation, connected with the inherently approximate nature of the meaning of Λ as the scale beyond which new physics emerges. Realistically, there is no such thing as a definite, abrupt cut-off. The transition to the new physics is gradual as we increase the energy of probes.

On the other hand, the uncertainties in the expression (50) for M_H^2 coming from the first term

(the square brackets) can be estimated to contribute up to 8 percent, and the uncertainties of the terms which are proportional to $\ln \Lambda$ in (50) contribute up to 5 percent. As argued just after eq. (50), the terms of $\mathcal{O}(\kappa g_t^2 \Lambda^2/\pi^2)$ would change M_H^2 by up to 10 percent (for $\Lambda < 1.5$ TeV). Altogether, this would result in an estimated uncertainty of 10-15 percent for M_H^2, and correspondingly 5-8 percent for M_H of Table 1 (sixth column). The uncertainty due to our ignoring the 2-loop contributions of scalars and gauge bosons to M_H appear to be substantially lower, even for the case of large $\lambda(\Lambda) \approx 3$.

4 Conclusions

We found out that in the minimal Standard Model, for a large subsector (51)-(52) of the possible values of the bare parameters $M^2(\Lambda)$ and $\lambda(\Lambda)$ in the tree level potential (3), the ultraviolet cut-off Λ of the theory should not be larger than $\mathcal{O}(1$ TeV), as long as we demand that the scalar self-interactions not be too strong, i.e., that they behave perturbatively. By the latter we mean that the 1-loop contributions of the scalar self-interaction to the (derivative of the) effective potential are taken to be distinctly smaller than those of the heavy quark Yukawa interaction. Furthermore, it turns out that the corresponding contributions of the electroweak gauge bosons are substantially smaller than those of the heavy top quark. The resulting Higgs masses are in the range 150-250 GeV.

The heavy top quark and the corresponding Λ^2-terms in the top-induced effective potential play a crucial role leading to the conclusions of the present paper. These effects of the heavy top quark sector on the scalar one are inherently non-perturbative. The case of the tree-level SSB ($M^2(\Lambda) > 0$) is just one part of the subsector (51) leading to $\Lambda \sim 1$ TeV. The other case of (51)-(52) where the conclusions of the paper apply is: no tree-level SSB, with massless or not very heavy scalar doublet at the tree level whose bare mass is $\mu(\Lambda) = \sqrt{-M^2(\Lambda)} \leq \mathcal{O}(v\sqrt{\lambda(\Lambda)})$ $(= \mathcal{O}(M_H))$. If the bare parameters $M^2(\Lambda)$ and $\lambda(\Lambda)$ do not fulfill any of the conditions (51)-(52), i.e., if $\mu(\Lambda) = \sqrt{-M^2(\Lambda)}$ is by at least an order of magnitude larger than $v\sqrt{\lambda(\Lambda)} = \mathcal{O}(M_H)$, then the cut-off Λ can become higher than $\mathcal{O}(1$ TeV), roughly of the order of $\mu(\Lambda)/\sqrt{\kappa} \sim 10\mu(\Lambda)$. The implications for the values of the cut-off Λ remain unclear if the scalar sector is strongly interacting, i.e., if the Higgs has a substantially larger mass than the values M_H listed in Table 1.

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5 Note added

After finishing the present work, a related work of T. Hambye came to our attention [11]. He discusses the case of the zero bare mass parameter $\mu^2(\Lambda) = 0$. We note that, in the present paper, the cut-off Λ acquires the values $\Lambda^{\text{u.b.}}$ of Table 1 precisely in that case. However, Hambye does not assume that the top quark loops necessarily dominate over the 1-loop scalar self-interaction effects in the effective potential. Thus, he allows for the values of the bare coupling $\lambda(\Lambda)$ a larger region which would be restricted only by the requirement that the scalar self-interaction be weak enough to be treated perturbatively. In the region where $\lambda(\Lambda) \leq 3$, his values for Λ and M_H agree roughly with the corresponding results of the present paper, i.e., with $\Lambda^{\text{u.b.}}(1lt + sc + gb)$ and $M_H(1lt + sc + gb)$ of Table 1, respectively. The small differences in numbers (for $\lambda(\Lambda) \leq 3$) arise largely from the fact that we took in account, in addition, the effects of the running of the various discussed parameters (g_t, m_t, φ, M_H) from the cut-off $E = \Lambda$ down to the electroweak energies.

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Table 1

$\lambda(\Lambda)$	$M_H^{(1\ell t)}$ [GeV]	$\Lambda^{\rm u.b.}(1\ell t)$ [TeV]	$M_H(1\ell t+sc)$ [GeV]	$\Lambda^{\rm u.b.}(1\ell t+sc)$ [TeV]	$M_H(1\ell t+sc+gb)$ [GeV]	$\Lambda^{\rm u.b.}(1\ell t+sc+gb)$ [TeV]
3.00	247	0.94	230	1.16 (51 %)	234	1.24
2.75	238	0.90	223	1.09 (46 %)	228	1.18
2.50	229	0.87	217	1.03 (42 %)	221	1.11
2.25	219	0.83	209	0.97 (38 %)	213	1.04
2.00	208	0.78	201	0.90 (34 %)	204	0.98
1.75	197	0.74	191	0.84 (29 %)	194	0.90
1.50	185	0.69	181	0.77 (25 %)	183	0.83
1.25	171	0.64	169	0.70 (21 %)	171	0.75
1.00	156	0.58	155	0.62 (17 %)	157	0.67

6 Table and figure captions

Table 1: The upper bounds for the ultraviolet cut-off Λ as a function of the physical Higgs mass, or alternatively, of the bare scalar self-interaction parameter $\lambda(\Lambda)$. The second and third column refer to the values when only the leading heavy top quark quantum contributions (non-perturbative) are included in the effective potential. The fourth and fifth columns are for the case when the leading scalar self-interaction quantum contributions (perturbative) are included; in the fifth column, we included in parentheses the percentages by which the upper bound for the square of the ultraviolet cut-off (Λ^2) changed with respect to the values of the third column. The sixth and seventh column are for the values when, in addition, also the leading electroweak gauge boson quantum contributions (perturbative) are included. We took $m_t^{\rm phy} = 180$ GeV, and always included also the leading logarithmic QCD correction to the running of the Yukawa coupling g_t between m_t and Λ.

Figures 1a-c: The 1-PI diagrams whose truncated Green functions lead to the 1-loop heavy quark contributions to the effective potential (calculated with the notation of unbroken fields).

Figures 2a-b: The diagrams whose truncated Green functions $\Sigma_{HH}^{tt}(q^2)$, $\Sigma_{HH}^{ZG}(q^2)$ and $\Sigma_{HH}^{WG}(q^2)$ contribute in the Landau gauge non-zero $\ln\Lambda$-terms to $[d\Sigma/dq^2|_{q^2=M_h^2}]$ and to $[\Sigma_{HH}(M_H^2) - \Sigma_{HH}(0)]$, and hence contribute to the renormalization of the field and the mass of the Higgs. The Goldstones are denoted as $G^{(0)}$ and $G^{(\pm)}$.

Figure 3: The tadpole contributions to the pole mass of the top quark, when 1-loop scalar self-interaction effects and gauge bosons are ignored. The mass in the tree-level propagators of the top quark is the bare mass $m_t^{(0)}(\Lambda)$; H is the Higgs field ($H = \varphi - \langle\varphi\rangle_{1\ell t}$); the second diagram is the contribution of the linear H-term.

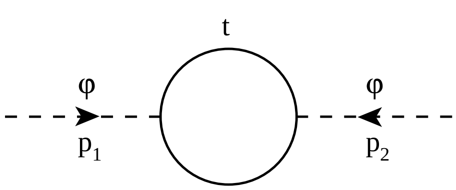

Fig. 1a

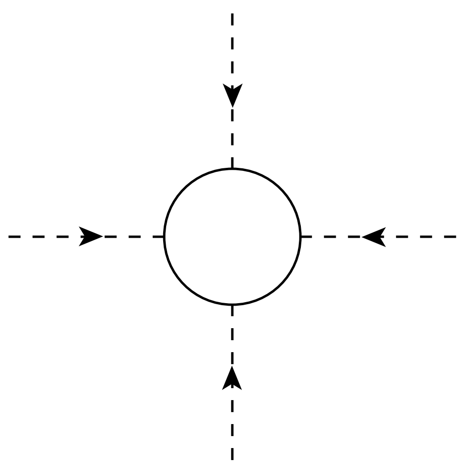

Fig. 1b

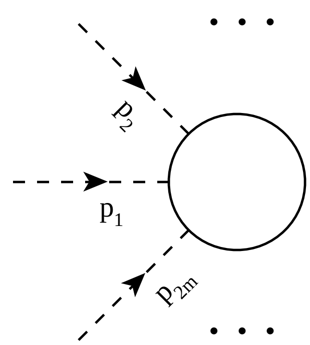

Fig. 1c

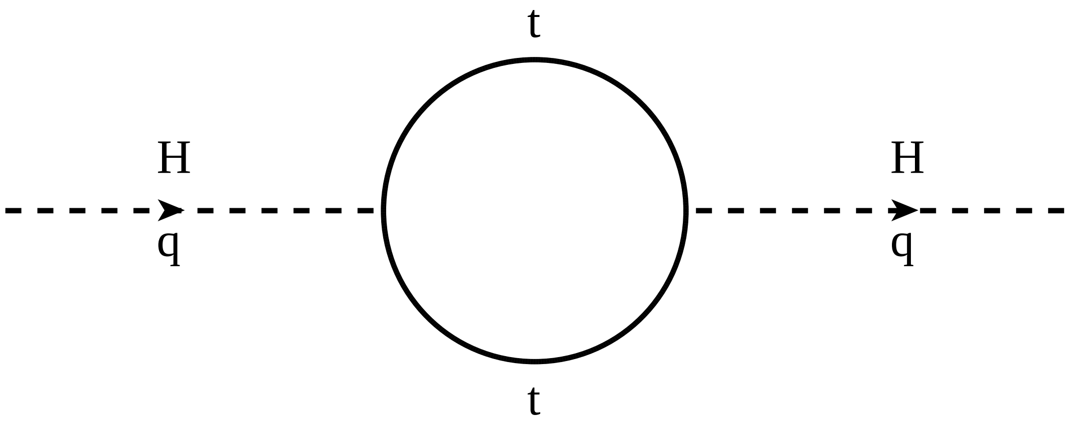

Fig. 2a

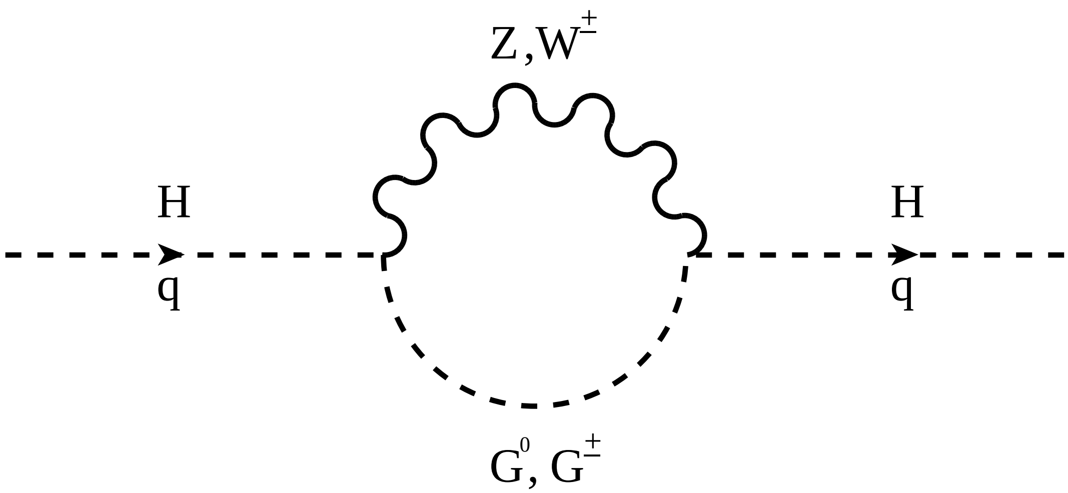

Fig. 2b

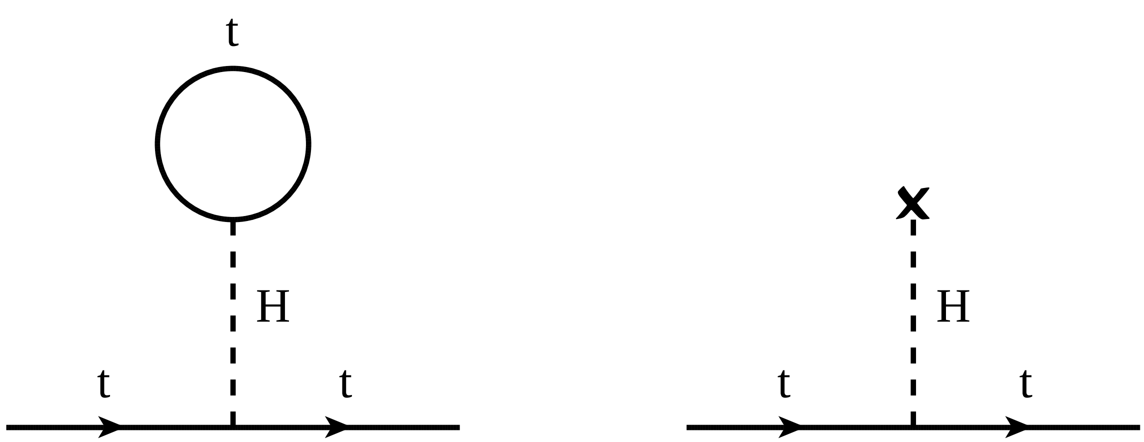

Figure 3